

Survival and growth of marked fish was normal during the test period. Whether growth interfered with mark retention was not investigated. Largemouth bass exhibited the highest growth rate and their reduced mark quality may suggest some correlation.

A disadvantage with the compressed air and fluorescent pigment technique is the required use of ultraviolet light for mark detection. However, in studies where the investigator desires exclusive knowledge of marked fish this character would be desirable. Also, detection rate is only 5-10 seconds per fish for a trained worker.

ACKNOWLEDGMENTS

The author wishes to acknowledge Mr. Wesley V. Fish, Florida Game and Fresh Water Fish Commission, for his assistance throughout the study. Thanks are also due to the personnel of the Richloam Fish Hatchery, Florida Game and Fresh Water Fish Commission, for their cooperation.

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CERTAIN ADVANTAGES OF SIMPLE FORMULATION IN FISH STUDIES AND STATISTICS

By Clark A. Ritchie

ABSTRACT

The growth of fish is an important factor in the useful studies of fish populations. There are several well known methods of comparing growth of fish. Only one will be discussed in this paper. This method uses the formula $W = KL^n$ where W is weight in grams, k is a constant, L is length in millimeters, and n is a power, usually near 3. This paper will espouse a variant of the formula, equating $W = KL^3$. Here, the variable n becomes a constant 3, eliminating the vagaries of n ; and the constant k now becomes a variable K changing with length, in order to maintain mathematical validity. K varies with L in this paper although it could be made to vary with W and to some approximate degree with age. It will be shown that the equation holds regardless of the size of the fish. The advantage of the simplified hyperbolic equation is that it reduces the input to three variables. So one variable which usually varies with length becomes the sole means of comparing the plumpness and condition of fish. This eliminates the fuzzy mathematical judgment involved when both changes in a constant and a power are involved in comparisons. It will be shown by illustration and example that this concept readily lends itself to simple single setting computer type solutions for K , L or W ; and to available tabular solutions in both English and metric systems. Thus, a method and three aids are proposed to decrease the effort and increase the reliability and usefulness in fisheries studies.

The world needs protein. Water provides the environment for production of such protein, and fish are a well known and highly acceptable source. Fish may well be the healthiest basic food. There are various yardsticks for measuring fish production, such as money value, tons of fish, and various breakdowns by fisheries. Age-weight

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and weight-length studies have an important bearing on the entire production outlook. Of less importance from a nutritional point of view, but still a significant contributor to food is sport fishing. This use of fisheries resource has an increasing and far reaching stimulus to the economy of the United States. Any useful method or improved tool which is readily accessible to the fishery biologist to reduce his work and improve his effectiveness will also be useful to the fisheries analysts, managers and administration.

A modified method which lends itself to such use as an effective and simple tool is the purpose of this paper. It is aimed specifically at the weight-length-condition factor phase.

The simplest general formula used in the weight-length relationship is $W = kL^n$ where W is weight in grams, k is a constant, L is length in millimeters and n is a power usually near 3. The English system version of this formula is $W = cL^n$ where W is weight in pounds, c is a constant, L is length in inches, and n in the English system equals n in the metric system. For convenience it is common practice to divide the right hand side of the equation by 100,000 or 10^5 in order to bring the constant near unity. The one important modification in this paper is that the power n will always equal 3, and in order to keep the equation valid, something must be changed or added. The author has chosen to make the constant k or c a variable which changes with length. The new variables will be K or C (capitals) where formerly they were k or c (lower case letters). For some few cases where the former n equalled 3, the new variables will be a constant but, as formerly, will usually change with the species. To reduce conversion errors on which more statistical information was available to the writer the English system formula will be used.

$$W = CL^3 \times 10^{-5}$$

Note particularly that C varies with length. Of course, it could be made to vary with weight and in some cases approximately with age. The writer claims no originality for the idea of holding $n = 3$ in the basic formula. However, he has not found in the literature the reasons set forth regarding the validity of the proposal and specific purpose to permit simple computations, comparisons, tabular solutions, and dial computer checks.

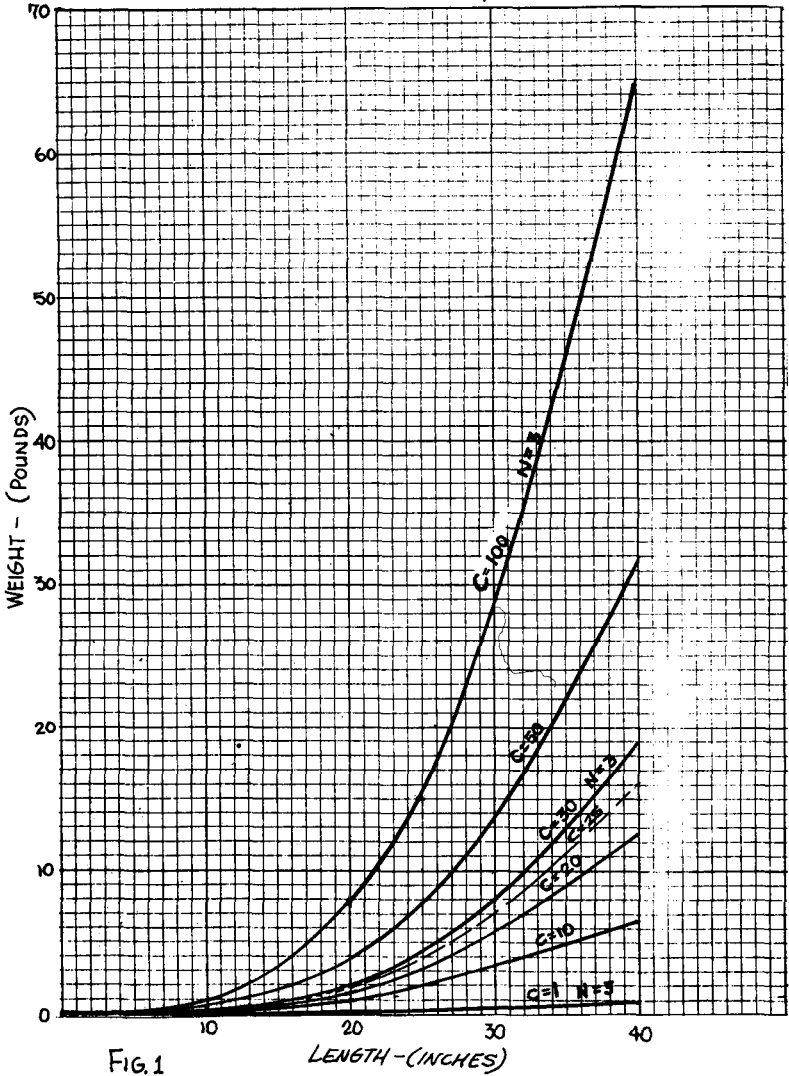
Validity of the Formula

Graphics will be presented to give a visual picture. Plot of the formula on linear graph paper, Figure 1, gives a family of third degree hyperbolic curves in common usage. For illustrative purposes C is varied in four steps as shown, $C = 1$, $C = 10$, $C = 50$, and $C = 100$. Note that 4 curves are formed all originating at $W = 0$, $L = 0$. Next these same curves are plotted on log log paper, Figure 2. Four parallel lines result, all with the same slope (3 to 1 because $n = 3$). On log log paper it is easy to visualize several things, viz.:

1. If one or more points are questionable, it is easier to identify such points.
2. If all points are not in a straight line, it is easier to pass a curve (straight line) through them, selecting a weighted average. This is particularly true at extreme values.
3. In the case where the points change from the 3rd degree curve, it is easy to describe the variation by using the new C value at that length.
4. The life cycle appears actually to be the locus of a point which changes during the yearly cycle usually on either side of the C line characteristic of the species, and with certain dramatic changes usually occurring, for instance, at the time of spawning or with a sudden large ingestion of food.
5. The slope (linear) of the curve on log log paper is the power n . It is obvious on log log paper that in order to be meaningful, the length of the curve should extend for adult fish from maturity to nearly old age. In the old method, the shorter the interval, the greater the error in n . In fact, n goes to + infinity with each ingestion, to zero with each defecation by the old method. One does not need to search the literature extensively to find an obvious difference or discrepancy in the value of n for the same species. By the method used herein, $n = 3$ and C changes to

$$W = CL^3 \times 10^{-5}$$

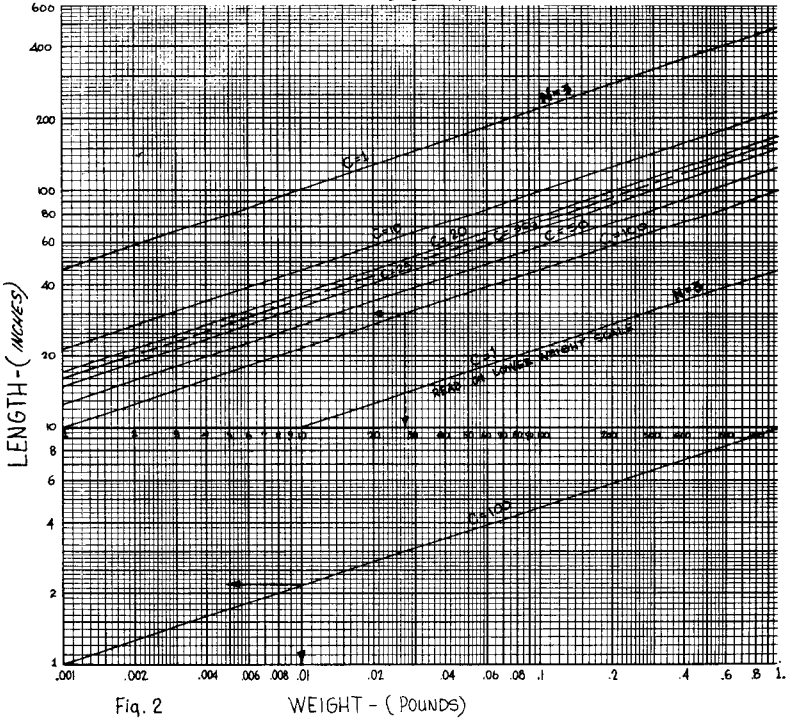
LINEAR GRAPH



accommodate the change in weight. By establishing that $n = 3$ throughout and for all fish, the comparison is limited to one variable, "C". For example, if a certain fish, Condition Factor $C = 30$, length 40 inches, weight 19.200 pounds, eats a 1.920 pound fish, his length does not immediately change; his weight goes to 21.12 pounds immediately, a 10% increase, and the C changes to 33, which is also a 10% increase. At the same length, the weights are proportional to the Condition Factors. To compare two different fish populations of the same species where both n and c vary

$$W = CL^3 \times 10^{-5}$$

LOG LOG GRAPH



(old method) it is necessary to calculate both formulae for every point of comparison; a time consuming and tedious chore, involving logarithmic calculation. By the method espoused herein where $n = 3$ and only C varies at each point of comparison, the weights will vary directly as C . C may sometimes be expressed by formula, or the notation may be empirical. In Figure 4 the C for Channel Catfish is approximately expressed by the formula, $C = 32 + .64(L - 15)$ where L is Length in Inches. This problem is readily solved by Weight Length, Condition Factor tables, a page of which is shown, Figure 3. The tables solve for W , L , or C when any 2 are known. The tables from which Figure 3 is an extract, cover fish from 0 to 109" in length, from 0 to 1295.03 pounds weight (5 place accuracy) and for condition factors 1 to 100. Weights are direct reading above 1 pound, and show the reciprocal of weight or number of fish per pound below 1 pound. To obtain actual weight of fingerlings and fry below one pound move the decimal for length one place to the right and the answer for weight three places to the left. Example; find the weight of a 6.1 inch fish, Condition Factor 30. Enter Condition Factor $C = 30$ table - find 61 inches length. Weight is 68.094 pounds. Move decimal three places to left. Answer .068094 pounds. Also, for fish longer than 109", move decimal place one place to left for length and move resulting answer for weight 3 places to right. Various table arrangements are available in both the English and metric systems. This method lends itself not only to fish, but to all water-suspended life forms from the smallest single celled organism to the largest whale. The principle holds for any magnitude and for any life form with constant density.

EXTRACT from RITCHIE'S FISH WEIGHT LENGTH CONDITION FACTOR TABLES*

Length Inches	Minus Sign Indicates Fish Per Pound									
	0	1	2	3	4	5	6	7	8	9
0	0.000	-3333.333	-416.667	-123.457	-52.083	-26.667	-15.432	09.718	-6.510	-4.572
10	-3.333	-2.504	-1.929	-1.517	-1.215	1.013	1.229	1.474	1.750	2.058
20	2.400	2.778	3.194	3.650	4.147	4.688	5.273	5.905	6.586	7.317
30	8.100	8.937	9.830	10.781	11.791	12.863	13.997	15.196	16.462	17.796
40	→19.200	20.676	22.226	23.852	25.555	27.338	29.201	31.147	33.178	35.295
50	37.500	39.795	42.182	44.663	47.239	49.913	52.685	55.558	58.534	61.614
60	64.800	→68.094	71.498	75.014	78.643	82.388	86.249	90.229	94.330	98.553
70	102.900	107.373	111.974	116.705	121.567	126.563	131.693	136.960	142.366	147.912
80	153.600	159.432	165.410	171.536	177.811	184.238	190.817	197.551	204.442	211.491
90	218.700	226.071	233.606	241.307	249.175	257.213	265.421	273.802	282.358	291.090
100	300.000	309.090	318.362	327.818	337.459	347.288	357.305	367.513	377.914	388.509

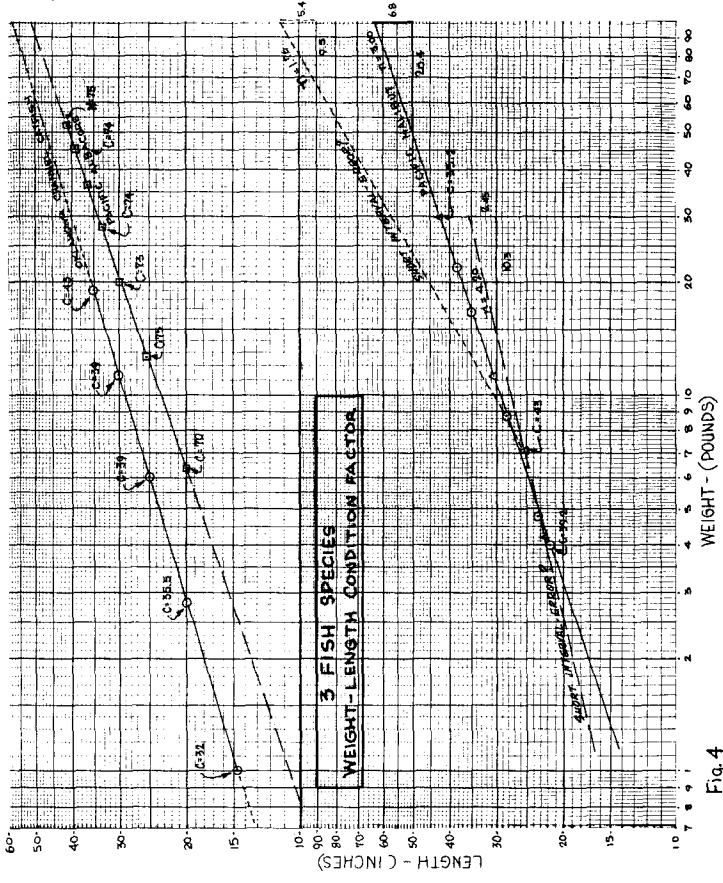
		Condition Factor is 30.0									
		0	1	2	3	4	5	6	7	8	9
0	0.000	-3030.303	-378.788	-112.233	-47.348	-24.242	-14.029	-8.835	-5.919	-4.157	
10	-3.030	-2.277	-1.754	-1.379	-1.104	1.114	1.352	1.621	1.925	2.263	
20	2.640	3.056	3.514	4.015	4.562	5.156	5.800	6.495	7.244	8.048	
30	8.910	9.831	10.813	11.859	12.970	14.149	15.396	16.715	18.108	19.575	
40	→21.120	22.744	24.449	26.237	28.111	30.071	32.121	34.262	36.495	38.824	
50	41.250	43.775	46.401	49.129	51.963	54.904	57.953	61.114	64.387	67.775	
60	71.280	74.904	78.648	82.516	86.508	90.626	94.874	99.252	103.763	108.408	
70	113.190	118.111	123.172	128.376	133.724	139.219	144.862	150.656	156.602	162.703	
80	168.960	175.376	181.951	188.690	195.592	202.661	209.898	217.306	224.886	232.640	
90	240.570	248.678	256.967	265.438	274.093	282.934	291.963	301.182	310.593	320.199	
100	330.000	339.999	350.199	360.600	371.205	382.016	393.035	404.264	415.705	427.360	

Direct reading metric tables are also available.

* All of Ritchie's Fish Weight Length Condition Factor Tables are Copyright 1968 by Clark A. Ritchie — all rights reserved.

Figure 3

REF: AVERAGE GROWTH RATE AND WEIGHT-LENGTH RELATIONSHIPS
 FOR FIFTY SPECIES OF FISH IN OKLAHOMA WATERS -
 PAGE 49 ALFRED HOUSER & MICHAEL G. BROSS REPORT NO 85, JAN 1963



REF: H.P. CLEMENS - 1961
 CALIF. DEPT OF
 FISH & GAME
 BULLETIN # 115 p.95

MALE Δ FEMALE Δ
 REF: THOMPSON & BELL
 INT. FISH COMM.
 RPT #8, 1934, p.25

Fig. 4

If n other than 3 is used, it must, in the writer's opinion in order to be useful and valid:

1. Cover the adult life span of the species.
2. Have the first and last observations occur at the same corresponding phase in the annual cyclic change.

3. Usually use two or more equations to fit the entire curve.

To reiterate, a simpler and more descriptive method of comparisons is in the use of the formula $W = CL^3 \times 10^{-5}$. Here, the weight-length relationship is governed by but one variable C which changes (usually) with length. Also, this formula is valid throughout the life span from the egg until old age. With the former method as reported by many writers, both n and c varied, which made for fuzzy comparisons and difficult calculations.

On log-log graph paper with the appropriate number of cycles and even of letterhead size, it is possible to trace the growth from the egg to old age with two place accuracy or better throughout. The dramatic change from the fertilized ovum where C = about 1900 for the fresh water fish egg (about 1950 for salt water eggs), diminishing fast to the C of the fry, fingerling, young adult, adult, and old age fish, may be thus graphically shown or described mathematically as Condition Factor varies with length. This will permit studies of fish comparisons, changes through the annual cycle and changes from year to year. It will also simplify comparative and analytical studies of specific species in any one body of water with those in any other body of water together with natural or controlled changes in food and all of the environmental factors and influences in the same or different habitats. Figure 4 illustrates a log log plot showing weight-length and Condition Factor of fish over part of the life span. Note that the slope of the line varies and changes erratically over different periods of observation with the greatest error in small increments of weight or length change.

Figure 5 illustrates a simple single setting circular computer which, with a turn of the dial, will solve with slide rule accuracy for:

1. Weight, with condition factor and length known.
2. Length, with condition factor and weight known.
3. Condition factor, with length and weight known.
4. K to C and C to K (same length basis).

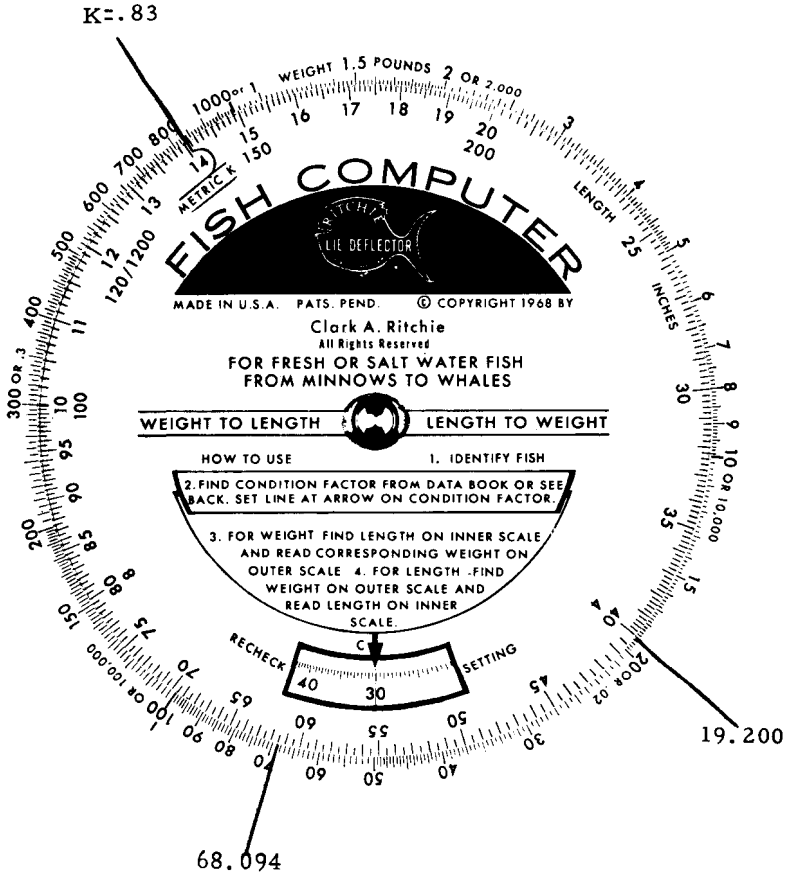
A model is available with both English and metric scales which, in addition to the above, will:

5. Give answers in both systems--pounds and kilograms and inches and centimeters simultaneously.
6. With an overlay, read dressed weight in either system.
7. Give fillet weight in either system.

SUMMARY

A method of computation is presented and illustrated graphically. The variable power formula requiring logarithmic solution to determine weight, length or condition factor is unwieldy and time-consuming. It is generally recognized in the literature that to follow the life span of the fish this formula usually requires variations. The formula which is valid for the life span of the fish is $W = CL^3 \times 10^{-5}$ where C varies with length. This formula is adaptable to one entry tabular solutions, and to simple, one setting dial computer solutions. Both the tables and the dial computer are available.

C = 30



(EDITOR'S NOTE: This item is available to the profession at \$4.95 each postpaid or \$50.00 per dozen.)

ANALOG COMPUTATION AND FISH POPULATION STUDIES¹

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The analog computer can give valuable assistance in the study of the dynamics of fish populations and has been used in yield analysis (Silliman, 1967), age and growth studies (Richards, 1968) and in studies of predator-prey relationships (Doi, 1962). In the near future, hybrid systems involving both the analog and digital computer will be used in the study of complete biological systems rather than isolated components. Mortality, fecundity, growth, intraspecific and interspecific relationships as they become known could be put together for system models. The purpose of this paper is to demonstrate some of the capability of the analog computer.

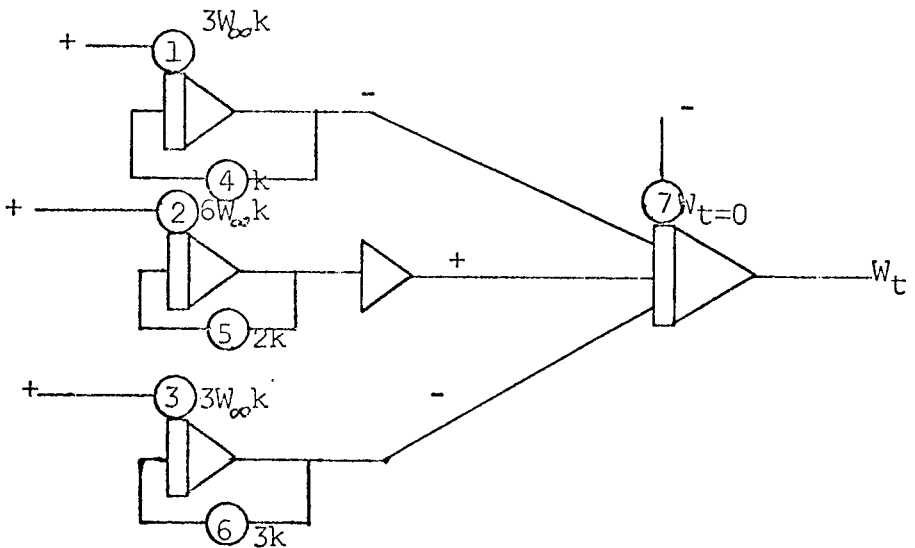
Segments or data blocks of a population study are growth in weight, mortality rates, and fecundity. These can be simulated by the analog computer. The expanded von Bertalanffy equation, where:

$$W_t = W_{\infty} - 3W_{\infty}e^{-kt} + 3W_{\infty}e^{-2kt} - W_{\infty}e^{-3kt} \quad (1)$$

is written as the differential equation:

$$\dot{W}_t = 3kW_{\infty}e^{-kt} - 6kW_{\infty}e^{-2kt} + 3kW_{\infty}e^{-3kt} \quad (2)$$

This equation is integrated and solved by the analog circuit:



Values of K and maximum weight are obtained through the technique described by Richards, (1968). Growth curves (Figs. 1-2) of female and male striped bass, (*Roccus saxatilis*), based on data from Mansueti (1961), are typically sigmoid.

¹Contribution No. 293 from Virginia Institute of Marine Science.